Synchronization in Oscillator Networks and Smart Grids

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References and Acknowledgments

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[1] F. Dörfler and F. Bullo. On the critical coupling for Kuramoto oscillators. *SIAM Journal on Applied Dynamical Systems*, 10(3):1070–1099, 2011

[2] F. Dörfler and F. Bullo. Synchronization and transient stability in power networks and non-uniform Kuramoto oscillators. *SIAM Journal on Control and Optimization*, 50(3):1616–1642, 2012

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Mark's early work on Synchronization

Nikhil and Mark identified the importance of synchronization in multi-agent systems for the control community in 2005.

In 2006, Mark gave a wonderful talk at TokyoTech and plenary at the IFAC Workshop on Lagrangian & Hamiltonian Methods in Nagoya.

Nikhil and Mark identified the connection between Kuramoto oscillators and consensus algorithms in proving expontial synchronization.

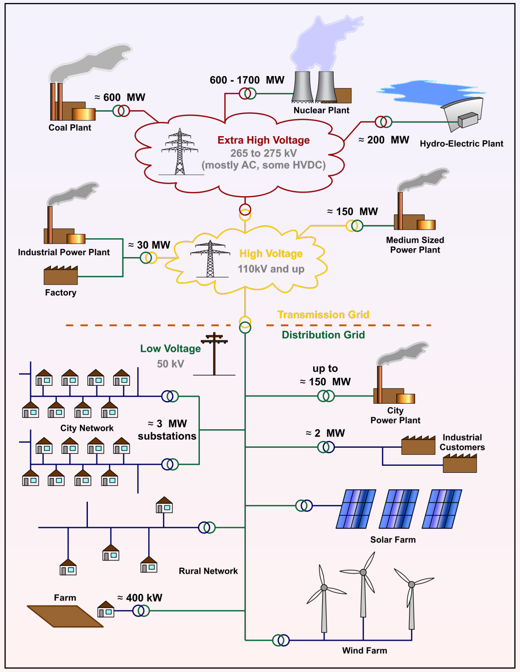
[1] N. Chopra and M. W. Spong. On synchronization of networked passive systems with time delays and application to bilateral teleoperation. In *Annual Conference of Society of Instrument and Control Engineers of Japan*, Okayama, Japan, 2005

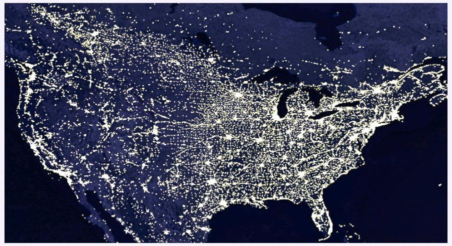
[2] M. W. Spong and N. Chopra. Synchronization of networked Lagrangian systems. In *Lagrangian and Hamiltonian Methods for Nonlinear Control 2006*, volume 366 of *Lecture Notes in Control and Information Sciences*, pages 47–59. Springer, 2007

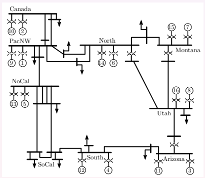
[3] N. Chopra and M. W. Spong. On exponential synchronization of Kuramoto oscillators. *IEEE Transactions on Automatic Control*, 54(2):353–357, 2009

Outline

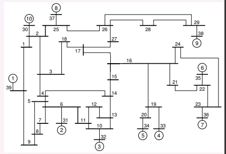
Power Generation and Transmission Network





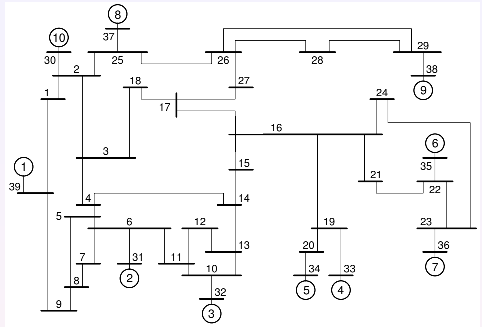


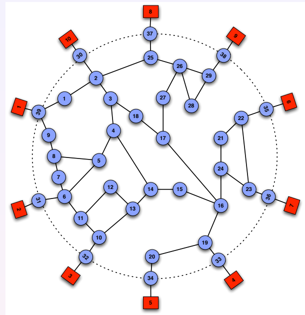
Western US
(WECC 16-m, 25-b)



New England
(10-m, 13-b)

Mathematical Model of a Power Transmission Network





1. n generators ■ and m load buses ●
2. admittance matrix $Y \in \mathbb{C}^{(n+m)\times(n+m)}$, symmetric, sparse, lossless

Central task: generators provide power for loads

Problems: monitoring and stability in face of disturbances and contingencies

Mathematical Model of a Power Transmission Network

1. power transfer on line $i \rightsquigarrow j$: $\underbrace{|V_i||V_j||Y_{ij}|}_{a_{ij}=\text{max power transfer}} \cdot \sin(\theta_i - \theta_j)$
2. power balance at node i: $\underbrace{P_i}_{\text{power injection}} = \sum_j a_{ij} \sin(\theta_i - \theta_j)$

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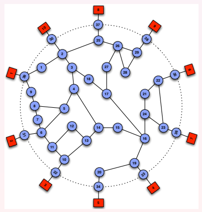
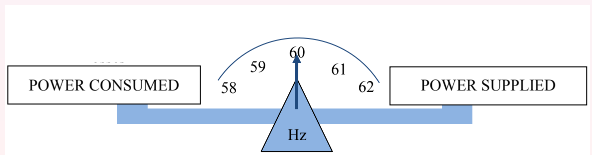
Structure-Preserving Model [Bergen & Hill '81]

for ■, swing eq with $P_i > 0$ $\quad M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$

for ●, const $P_i < 0$ and $D_i \geq 0$ $\quad D_i\dot{\theta}_i = P_i - \sum_j a_{ij}\sin(\theta_i - \theta_j)$





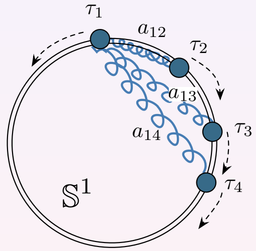
Synchronization in Power Networks

1. power networks are coupled oscillators

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum\nolimits_j a_{ij}\sin(\theta_i - \theta_j)$$

$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j a_{ij}\sin(\theta_i - \theta_j)$$

2. synchronization: coupling strength vs. frequency non-uniformity



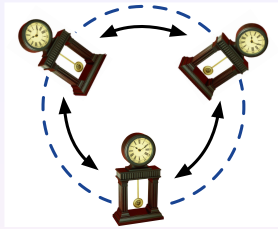
3. graph theory: "coupling/connectivity" and "non-uniformity"

power networks **should** synchronize
for large "coupling/connectivity" and small "non-uniformity"

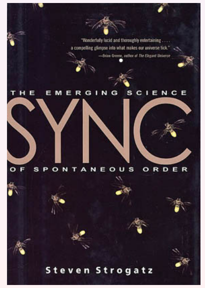
Coupled Oscillators in Science and Technology

Kuramoto model of coupled oscillators:

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j)$$



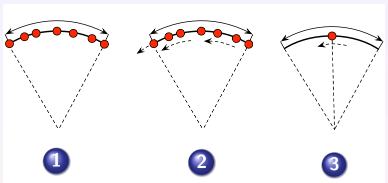
- Sync in Josephson junctions [S. Watanabe et. al '97, K. Wiesenfeld et al. '98]
- Sync in a population of fireflies [G.B. Ermentrout '90, Y. Zhou et al. '06]
- Coordination of particle models [R. Sepulchre et al. '07, D. Klein et al. '09]
- Deep-brain stimulation and neuroscience [P.A. Tass '03, E. Brown et al. '04]
- Countless other sync phenomena [A. Winfree '67, S.H. Strogatz '00, J. Acebrón '01]



Synchronization Notions

$$\dot{\theta}_i = \omega_i - \sum\nolimits_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j)$$

1. phase cohesive: $|\theta_i(t) - \theta_j(t)| < \gamma$ for small $\gamma < \pi/2$ ... arc invariance
2. frequency synchrony: $\dot{\theta}_i(t) = \dot{\theta}_j(t)$
3. phase synchrony: $\theta_i(t) = \theta_j(t)$



- $\{a_{ij}\}_{\{i,j\}\in\mathcal{E}}$ small & $|\omega_i - \omega_j|$ large $\Longrightarrow$ no synchronization
- $\{a_{ij}\}_{\{i,j\}\in\mathcal{E}}$ large & $|\omega_i - \omega_j|$ small $\Longrightarrow$ cohesive + freq sync

Challenge: proper notions of sync, coupling & phase transition
[A. Jadbabaie et al. '04, P. Monzon et al. '06, Sepulchre et al. '07, S.J. Chung et al. '10, J.L. van Hemmen et al. '93, F. de Smet et al. '07, N. Chopra et al. '09, G. Schmidt et al. '09, F. Dörfler et al. '09 & '11, S.J. Chung et al. '10, A. Franci et al. '10, S.Y. Ha et al. '10, D. Aeyels et al. '04, R.E. Mirollo et al. '05, M. Verwoerd et al. '08, L. DeVille '11, . . .]

Outline

Primer on Algebraic Graph Theory

Graph: weights $a_{ij} > 0$ on edges $\{i,j\}$, values x_i at nodes i

- adjacency matrix $A = (a_{ij})$
- degree matrix D is diagonal with $d_{ii} = \sum_{j=1}^{n} a_{ij}$
- Laplacian matrix $L = L^T = D - A \geq 0$
  (pseudo-inverse of L = same eigenvectors, inverse eigenvalues)

Notions of Connectivity

topological: connectivity, average and worst-case path lengths
spectral: second smallest eigenvalue λ_2 of L is "algebraic connectivity"

Notions of Dissimilarity

$\|x\|_{\infty,\text{edges}} = \max_{\{i,j\}} |x_i - x_j|$, $\quad \|x\|_{2,\text{edges}} = \left(\sum_{\{i,j\}} |x_i - x_j|^2\right)^{1/2}$

(graph edges $\{i,j\} \in \mathcal{E}$) or (all edges $\{i,j\}$ satisfy $i < j$)

Sync Tests: Coupling vs. Power Imbalance

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum\nolimits_j a_{ij} \sin(\theta_i - \theta_j)$$
$$D_i \dot{\theta}_i = P_i - \sum\nolimits_j a_{ij} \sin(\theta_i - \theta_j)$$

$\sum_j a_{ij} \leq |P_i| \implies$ no sync $\qquad \lambda_2(L) > \|P\|_{2,\text{all edges}} \implies$ sync

Valid for: completely arbitrary weighted connected graphs

$$\left\|L^\dagger P\right\|_{\infty,\text{graph edges}} < 1 \iff \text{sync}$$

Correct for: trees, graphs with disjoint 3- and 4-cycles
Correct for: graphs with $L^\dagger P$ bipolar or symmetric
Correct for :* homogeneous graphs ($a_{ij} = K > 0$)

best general conditions known to date

A Nearly Exact Synchronization Condition – Accuracy

Randomized power network test cases
with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Correctness of condition: $\|L^\dagger P\|_{\infty,\text{g. edges}} \leq \sin(\gamma)$ $\Rightarrow \max_{\{i,j\}\in\mathcal{E}} \|\theta_i^* - \theta_j^*\| \leq \gamma$	Accuracy of condition: $\max_{\{i,j\}} \|\theta_i^* - \theta_j^*\|$ $- \arcsin(\|B^T L^\dagger P\|_\infty)$	Phase cohesiveness: $\max_{\{i,j\}\in\mathcal{E}} \|\theta_i^* - \theta_j^*\|$
9 bus system	always true	$4.1218 \cdot 10^{-5}$ rad	0.12889 rad
IEEE 14 bus system	always true	$2.7995 \cdot 10^{-4}$ rad	0.16622 rad
IEEE RTS 24	always true	$1.7089 \cdot 10^{-3}$ rad	0.22309 rad
IEEE 30 bus system	always true	$2.6140 \cdot 10^{-4}$ rad	0.1643 rad
New England 39	always true	$6.6355 \cdot 10^{-5}$ rad	0.16821 rad
IEEE 57 bus system	always true	$2.0630 \cdot 10^{-2}$ rad	0.20295 rad
IEEE RTS 96	always true	$2.6076 \cdot 10^{-3}$ rad	0.24593 rad
IEEE 118 bus system	always true	$5.9959 \cdot 10^{-4}$ rad	0.23524 rad
IEEE 300 bus system	always true	$5.2618 \cdot 10^{-4}$ rad	0.43204 rad
Polish 2383 bus system (winter peak 1999/2000)	always true	$4.2183 \cdot 10^{-3}$ rad	0.25144 rad

condition $\left\|L^\dagger P\right\|_{\infty,\text{graph edges}} \leq \sin(\gamma)$ is extremely accurate

Outline

Synchronization in a All-to-All Homogeneous Graph

all-to-all homogeneous graph $\dot{\theta}_i = \omega_i - \frac{K}{n}\sum_{j=1}^{n} \sin(\theta_i - \theta_j)$

Explicit, necessary, and sufficient condition [F. Dörfler & F. Bullo '10]

Following statements are equivalent:

1. Coupling dominates non-uniformity, i.e., $K > K_{\text{critical}} \triangleq \omega_{\max} - \omega_{\min}$
2. Kuramoto models with $\{\omega_1, \dots, \omega_n\} \subseteq [\omega_{\min}, \omega_{\max}]$ achieve phase cohesiveness & exponential frequency synchronization

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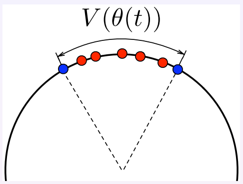
Define $\gamma_{\min}$ & $\gamma_{\max}$ by $K_{\text{critical}}/K = \sin(\gamma_{\min}) = \sin(\gamma_{\max})$, then

1) **phase cohesiveness** for all arc-lengths $\gamma \in [\gamma_{\min}, \gamma_{\max}]$
2) **practical phase synchronization:** from $\gamma_{\max}$ arc $\to$ $\gamma_{\min}$ arc
3) exponential **frequency synchronization** in the interior of $\gamma_{\max}$ arc

Main proof ideas (Nikhil's and Mark's work)

1. **Cohesiveness:**



- for $\theta(0)$ in arc of length $\gamma \in [\gamma_{\min}, \gamma_{\max}]$, define arc-length cost function

$$V(\theta(t)) = \max\{|\theta_i(t) - \theta_j(t)|\}_{i,j \in \{1,\dots,n\}}$$

- $t \mapsto V(\theta(t))$ is non-increasing because

$$D^+ V(\theta(t)) < 0$$

- $t \mapsto \theta(t)$ remains in (possibly-rotating) arc of length γ and, moreover, $\gamma < \pi/2$ in finite time

2. **Frequency synchronization:** once in arc of length $\pi/2$

$$\frac{d}{dt}\dot{\theta}_i = -\sum_{j \neq i} a_{ij}(t)(\dot{\theta}_i - \dot{\theta}_j)$$

where $a_{ij}(t) = \frac{K}{n}\cos(\theta_i(t) - \theta_j(t)) > 0$. result follows from time-varying consensus theorem

Outline

Conclusions

Summary:

1. connection between power networks and coupled Kuramoto oscillators
2. necessary and sufficient sync conditions

Ongoing and future work:

1. more realistic models: active+reactive power flow
2. sharp condition: tests and proofs
3. region of attraction
4. smart-grid apps = remedial action, wide-area control

IEEE CDC '12: Tutorial Session on Coupled Oscillators

F. Dörfler and F. Bullo. Exploring synchronization in complex oscillator networks. In *IEEE Conf. on Decision and Control*, Maui, HI, USA, December 2012. To appear